1-D Elastic Collisions

Conservation of momentum means that the total momentum in any type of interaction will be conserved. Naturally, this includes when two objects collide with each other. Typically, the only thing one can say about a collision is that momentum is conserved, but there is a special type of collision in which the total kinetic energy is also conserved. This can only happen if the kinetic energy of the objects turns into a potential energy at the start of the collision, so that it can then be turned back into kinetic energy by the end of the collision. In lab, we saw this with the magnets at the ends of the carts and if we are making up physics problems, it could be an ideal spring at the end of a cart.

Elastic Collision Kinetic energy is also conserved. (Bouncing "perfectly.")

Inelastic Collision Kinetic energy is <u>not</u> conserved. (Just bouncing.)

Completely Inelastic Collision When two objects end up sticking together. (Not bouncing.)

To be clear, energy is also always conserved. In an inelastic collision, some of the kinetic energy turns into thermal energy, and perhaps sound or other forms of energy, the total kinetic energy after the collision is less than it was before the collision.

General Equation Derivation: Elastic Collision in One Dimension

Imagine we have two objects with two initial velocities as shown. In that diagram, they will collide if $v_{1i} > v_{2i}$ or if v_{2i} is negative. (And if we flip the order in the diagram, we flip the order of those conditions.) If the collision between the two masses is one dimensional, and also elastic, we can find the final velocities of the two masses.

Momentum is always conserved, so we can start off by saying:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Because the collision is elastic, we can also say:

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

We have two equations with two unkowns. It should be easy to solve for the final velocities, but notice how annoying that will be. It is straight forward to take the momentum equation and solve for v_{1f} or v_{2f}, but when we go to substitute that into the kinetic energy equation we will have to square it, which will make a huge algebra mess. Thankfully, there is an eaiser way: regroup the equations so that all the m_1 terms are on the left of the equal sign and all the m_2 terms are on the right.

Conservation of momentum becomes:

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f} - m_2 v_{2i}$$

And conservation of kinetic energy becomes:

$$m_1 v_{1i}^2 - m_1 v_{1f}^2 = m_2 v_{2f}^2 - m_2 v_{2i}^2$$

Now comes the algebra fun. Divide those two equation as follows:

$$\frac{m_1 v_{1i}^2 - m_1 v_{1f}^2}{m_1 v_{1i} - m_1 v_{1f}} = \frac{m_2 v_{2f}^2 - m_2 v_{2i}^2}{m_2 v_{2f} - m_2 v_{2i}}$$

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Which reduces to

$$\boldsymbol{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

We will still solve this for v_{1f} or v_{2f}, but the huge advantage is that we can choose where to substitute this since it came from both of the original equations. So we will solve for v_{2f} and then plug that into the momentum equation as follows:

$$v_{2f} = v_{1i} + v_{1f} - v_{2i}$$

So that

$$m_1v_{1i} - m_1v_{1f} = m_2(v_{1i} + v_{1f} - v_{2i}) - m_2v_{2i}$$

Finally, just do the algebra to solve for v_{1f}.

$$m_1v_{1i} - m_1v_{1f} = m_2v_{1i} + m_2v_{1f} - m_2v_{2i} - m_2v_{2i}$$
$$-m_1v_{1f} - m_2v_{1f} = m_2v_{1i} - m_2v_{2i} - m_2v_{2i} - m_1v_{1i}$$
$$(m_1 + m_2)v_{1f} = m_1v_{1i} - m_2v_{1i} + m_2v_{2i} + m_2v_{2i}$$
$$(m_1 + m_2)v_{1f} = (m_1 - m_2)v_{1i} + 2m_2v_{2i}$$

Until we finally get:

$$\boldsymbol{v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)}v_{1i} + \frac{2m_2}{(m_1 + m_2)}v_{2i}}$$

Just redo all that algebra, this time solving our simplified velocity equation for v_{1f} and plugging that into the momentum equation. (Let's leave that as an exercise for the reader.) After doing that, we get

$$\boldsymbol{v_{2f} = \frac{2m_1}{(m_1 + m_2)}v_{1i} + \frac{(m_2 - m_1)}{(m_1 + m_2)}v_{2i}}$$

Please keep in mind that those two equations are <u>very</u> specific: they only apply to one-dimensional, elastic collisions. There are some interesting special cases of these types of collisions which we will look at in a follow up worksheet.

It turns out that there is something remarkably simple (and surprising) that happens in a one-dimensional, elastic collision. Recall the expression from the top of the page

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

This can be rewritten as

$$\boldsymbol{v_{1i} - v_{2i} = v_{2f} - v_{1f}}$$

The difference of the initial velocities is opposite the difference of the final velocities! This can be seen in the Logger Pro graph below: